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2.2.2 Network Tomography

A related problem is described in Vardi [1996] and Tebaldi and West [1998]. Called "network tomography" by Vardi, the idea is to infer the traffic intensity across the routes in a network using only measurements of intensity at the nodes.

We start with a network of nodes with directed connections between the nodes. In graph theory, this is called a directed graph, or digraph. Traffic can travel from one node to another if and only if there is a connection from the first to the second. This is illustrated in Figure 2.1. In this figure, there are two possible routes from a to b: $a \to b$ and $a \to c \to b$ (while the route $a \to c \to d \to c \to b$ is theoretically

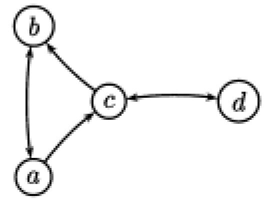

Fig. 2.1 A simple four-node network with directed arcs indicating links between nodes.

possible, we do not allow traffic to retrace its steps in this model). There is a single route possible from b to c: $b \to a \to c$.

Assume a network of n nodes. Then there are $c = n(n-1)$ source/destination (SD) pairs. Let r denote the number of directed links. In Figure 2.1, $n = 4$, $c = 12$, and $r = 6$.

The work proceeds along two lines. First, in the simpler case, the routes are fixed and known. In the second case, the routes are random but the transition probabilities between nodes are known. We will consider the fixed route in this section.

In the first case, define the routing matrix A as an $r \times c$ binary matrix, where $a_{ij} = 1$ if and only if the ith link is in the route between the jth SD pair. This is illustrated in Table 2.1. Note that although it is possible to go between a and b via the node c, the routing matrix indicates that this route is not allowed (see the first column of the routing matrix). Only the direct link from $a \to b$ is allowed in the SD pair ab, so the routing matrix provides the information necessary to decide the route between any two nodes, and these routes are unique.

For a set of measurement periods, we will measure the traffic along each directed link. These observations are denoted $Y^{(k)} = (Y^{(k)}{}_1, ..., Y_r^{(k)})$, taken at times $k = 1, ..., K$. The underlying random variables are $X^{(k)} = (X_1^{(k)}, ..., X_c^{(k)})$,

Table 2.1 A routing matrix for routes for the network in Figure 2.1. The columns correspond to source/destination (SD) pairs, while the rows correspond to the links between nodes. Thus, a 1 in position i, j indicates that link i is used in the route corresponding to source/destination (SD) pair j. A blank corresponds to a 0, indicating that the link does not appear in the route.

	ab	ac	ad	ba	bc	bd	ca	cb	cd	da	db	dc
$a \to b$	1											
$a \to c$		1	1		1	1						
$b \to a$				1	1	1	1			1		
$c \to b$							1	1		1	1	
$c \to d$			1			1			1			
$d \to c$										1	1	1

where each $X_j^{(k)}$ is the number of transmitted messages for the SD pair j at measurement period k. Thus,

$$Y^{(k)} = X^{(k)} A. \tag{2.8}$$

We assume that the $X_j^{(k)}$ are independent for each j and k, and distributed as a Poisson distribution, with a different rate for each SD pair. In other words, $X_j^{(k)} \sim Poisson(\lambda_j)$, where the Poisson distribution is defined as

$$f(x; \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, \tag{2.9}$$

where x is constrained to be a nonnegative integer. The mean and variance of a Poisson random variable with density as in Equation (2.9) are both λ. For more information about the Poisson distribution, see any statistics text, such as Hogg and Craig [1995].

Thus, the goal is to estimate $\lambda = (\lambda_1, ..., \lambda_c)$ from the observations $Y^{(k)}$. First we must determine whether we can in fact estimate λ.

Recall that a parameter vector is identifiable if it can be determined uniquely. For example, if we can only determine $\lambda_1 + \lambda_2$, we cannot "identify" the vector (λ_1, λ_2). For another example of nonidentifiability, consider the problem of trying to identify a mixture of two uniform densities. Consider Figure 2.2. This could be modeled as a mixture of uniform densities in many ways, in particular

$$\frac{1}{4}U\left(0, \frac{1}{2}\right) + \frac{3}{4}U\left(\frac{1}{2}, 1\right), \tag{2.10}$$

$$\frac{1}{2}U(0,1) + \frac{1}{2}U\left(\frac{1}{2}, 1\right). \tag{2.11}$$

There is no way to distinguish the parameters in these models since the different parameters produce the same density.

In the Poisson model we are considering here, λ is only identifiable when all the columns of A contain at least one nonzero entry and are distinct. Clearly, real-life routing matrices would generally not have a column of zeros (this corresponds to a source/destination pair that has no route: "you can't get there from here"). An example of identical columns can be found by considering Figure 2.1 and allowing the following two routes:

$ab : a \to c \to b \to a \to b$

and

$ac : a \to b \to a \to c \to b \to a \to c,$

which correspond to two columns equal to $(111100)^T$. This also is clearly not likely in a real routing matrix due to the number of backtracks. Clearly, these routes are not possible in real life since they both would stop as soon as the destination node had been reached.